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HEAT TRANSFER IN CONDENSATION AND EVAPORATION

MODELING OF GAS COOLING AND CONDENSATION IN A PISTON EXPANSION MACHINE

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A model of gas cooling and condensation in a piston expansion machine has been developed with allowance made for formation of nuclei of the liquid phase and growth of droplets as well as for real thermophysical properties. Internal heat release related to phase transition is taken into account in the adiabatic equation. The processes of cooling and liquefaction of methane in the piston expander are calculated for specific conditions.

A method using pre-cooling with expansion in the gas-expansion machine (or machines) and cooling due to the Joule–Thomson effect with pressure drop in the throttle is widely used for gas liquefaction [1, 2]. In the present work, we suggest a mathematical model of gas cooling and liquefaction in the piston expansion machine. Kuznetsov and Gorbenko [3] calculated cooling of air, helium, and methane with account for real physical properties and estimated the effect of friction and heat fluxes through the walls of the cylinder of the gas-expansion machine. In [3], such initial parameters are taken at which gas supersaturation is not reached and condensation is absent. In principle, partial condensation (liquefaction) is possible in gas-expansion machines. In the developed computational model, supersaturation in the gas-expansion machine is monitored and if it reaches a critical value, then condensation and corresponding heat release are taken into account. Usually it is assumed that in gas-expansion machines condensation is intolerable due to possible hydraulic shocks (this danger is impossible in small liquefaction) and difficulties with lubrication. Modern technologies allow solution of this problem.

In order to construct the computational model we adopted the following assumptions: the cylinder of the gas-expansion machine is adiabatically insulated and the process of gas expansion in it is isentropic until limiting supersaturation is reached; pressure and temperature are equalized in the expanded volume, friction and heat fluxes from the outside are absent. We use a zone method, i.e., the entire process of gas expansion in the gas-expansion machine (current time) is divided into time intervals (steps) $\Delta\tau_i$, to which space zones behind the piston moving in the cylinder will correspond. One time step is equal to $\Delta\tau_i = \tau_{2i} - \tau_{1i}$ and the increment of the zone volume is $\Delta V_i = V_{2i} - V_{1i}$. The medium parameters at the beginning of each step correspond to the parameters at the end of the previous step. Since, under the expansion conditions in the gas-expansion machine, the gas differs in its thermodynamic and thermal properties from an ideal gas, the adiabatic index $k = C_p/C_V$ depends on temperature and pressure. It was assumed that the general adiabatic equation retains its form within narrow limits of temperature and pressure variation with substitution of the adiabatic index by a value of k_{eff} that is effective in these conditions and that corresponds to the properties of the real gas. Thus, for the indicated narrow sections the equation for the adiabat takes on the form

$$T_{2i}/T_{1i} = (p_{2i}/p_{1i})^{A_{\text{eff}}} , \tag{1}$$

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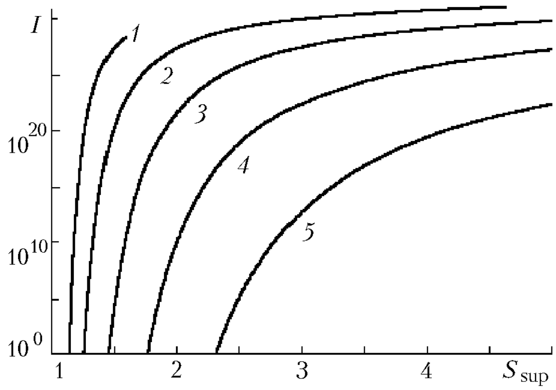

Fig. 1. Dependence of the nucleation rate I on the value of methane vapor supersaturation S_{sup} at different temperatures (calculation by the Frenkel' formula): 1, 2, 3, 4, and 5) 150, 140, 130, 120, and 110 K, respectively.

where $A_{eff}(T, p) = (k_{eff} - 1)/k_{eff}$ provided that $(T_{1i} - T_{2i})/T_{1i} << 1$ and $(p_{1i} - p_{2i})/p_{1i} << 1$. A_{eff} values can be related to the inlet temperature T_{1i} and pressure p_{1i} or, for example, to section-mean values of T and p. The values of $A_{eff}(T, p)$ for methane on the adiabats are determined from the equation of state [4] and the relationship

$$A_{eff} = \ln (T_{2i}/T_{1i})/\ln (p_{2i}/p_{1i}) . \tag{2}$$

The values of A_{eff} obtained for methane at p/p_s values from 0.8 to 1 change within the limits 0.285–0.275 as the entropy increases from 3.6 to 4.4 kJ/(kg·K), i.e., within the considered operating range of methane entropy values. Variation of A_{eff} on the adiabats within the indicated range of p/p_s does not exceed 0.03 on the average. For supersaturated vapor, in calculation of adiabats in the T and p parameters we take approximately that A_{eff} values remain the same as they were in crossing of the boundary curve by this adiabat.

The intensity of nucleation of the liquid phase in supersaturated vapor was calculated by the known Frenkel' formula [5], which gives, as was shown in [6], better correspondence to experimental data for water vapor than other available formulas. As critical vapor supersaturation $S_{sup,cr}$ we took such supersaturation S_{sup} at which the nucleation rate $I = 1$ cm^{-3}·sec^{-1} is reached. As S_{sup} increases further, the value of I increases sharply (Fig. 1). In calculation of I the thermophysical properties of methane were used [4, 7, 8].

Pressure (elasticity) of saturated vapor above the surface of a spherical nucleus (droplet) is determined by the Kelvin formula

$$p_{s\delta} (T_d, \delta) = p_{s,f} (T_d) \exp \left(\frac{4\sigma\mu}{RT_d \rho_{liq} \delta} \right). \tag{3}$$

Hence, at $S_{sup} = p_{s\delta}/p_{s,f}$ we find the nucleus diameter

$$\delta = \delta_n = \frac{1}{\ln S_{sup}} \frac{4\sigma\mu}{RT\rho_{liq}} . \tag{4}$$

The critical diameter of a nucleus $\delta_{n,cr}$ corresponds to critical supersaturation $S_{sup,cr}$. At $T = 140$ K and $I = 1$ cm^{-3}·sec^{-1}, $S_{sup,cr} = 1.263$ and $\delta_{n,cr} = 43.8 \cdot 10^{-8}$ cm = 43.8 Å. For larger droplets the ratio $p_{s\delta}/p_{s,f}$ is smaller. For methane droplets with a diameter $\delta = 5 \cdot 10^{-6}$ cm the considered ratio is of about 1.02; therefore, for these and larger droplets an increase in vapor elasticity due to surface curvature can be neglected.

Large droplets have a higher temperature. As the droplet diameter decreases, its temperature decreases and differs slightly (by tenths of a degree) from the surrounding temperature. The nucleus temperature was taken to be equal to the temperature of vapor. The values of δ_n for methane, depending on supersaturation at temperature 140 K, are as follows:

TABLE 1. Mean-Free Path of Methane Molecules and Knudsen Number at Different Parameters of State and Diameters of Droplets

T, K	p, Pa	p_s, Pa	$\bar{l}$, m	Kn		
				1	2	3
115	$2.2 \cdot 10^5$	$1.32 \cdot 10^5$	$8.8 \cdot 10^{-9}$	0.035	0.35	3.5
120	$3 \cdot 10^5$	$1.91 \cdot 10^5$	$6.7 \cdot 10^{-9}$	0.027	0.27	2.7
126	$5 \cdot 10^5$	$2.87 \cdot 10^5$	$4.2 \cdot 10^{-9}$	0.017	0.17	1.7
128	$6 \cdot 10^5$	$3.25 \cdot 10^5$	$3.6 \cdot 10^{-9}$	0.014	0.14	1.4
138	$8 \cdot 10^5$	$5.77 \cdot 10^5$	$2.9 \cdot 10^{-9}$	0.012	0.12	1.2

Note. 1, 2, and 3 at $\delta = 5 \cdot 10^{-7}$, $5 \cdot 10^{-8}$, and $5 \cdot 10^{-9}$ m, respectively.

S_{sup}	1.2	$S_{\text{sup,cr}} = 1.263$	1.3	1.4	1.5	2	2.5
$\delta_n \cdot 10^{10}$, m	56.1	43.8	39.0	30.4	25.2	14.8	10.7

The diameter of large droplets is large compared with the mean-free path of vapor molecules. At the initial stage of growth of droplets, their diameters are smaller than the mean-free path.

By supersaturation above large droplets (over a flat surface) we understand the ratio of vapor pressure p_∞ to saturated vapor pressure at temperature T_∞, i.e., $S_{\text{sup}} = p_\infty / p_{s\infty}$. Near the surface of a large droplet the pressure is equal to the pressure of saturated vapor at temperature $T_d > T_\infty$. For small droplets $T_d \approx T_\infty$, $p_d \approx p_\infty$ (due to the surface-curvature effect). As the process develops, vapor supersaturation changes with temperature, pressure, nucleation rate, and growth of droplets.

We dwell on direct account for the effect of the mean-free path $\bar{l}$ to the droplet radius, i.e., the Knudsen number Kn = $\bar{l}/(\delta/2)$, on the process under consideration. We estimate the mean-free path of the methane molecules as follows:

$$\bar{l} = \frac{1}{\sqrt{2}\,\pi d_m^2 n} = \frac{k_B T}{\sqrt{2}\,\pi d_m^2 p} . \tag{5}$$

To find d_m we use the rounded-off value of the parameter σ (σ = 3.8 Å) of the Lennard-Jones potential for methane given in [9]. The minimum energy in the Lennard-Jones potential measures up to the distance $\sqrt[6]{2}\,\sigma = 1.12\sigma$, which we take as the molecule diameter. Thus, for methane $d_m = 1.12\sigma = 4.3 \cdot 10^{-10}$ m in the then-used model where molecules are considered as solid spheres. The calculated values of $\bar{l}$ for supersaturated methane vapor at several temperatures and pressures are listed in Table 1 (calculation by the same technique for nitrogen under normal conditions gives $\bar{l} \approx 5 \cdot 10^{-8}$ m). This table also gives the Knudsen number values.

We estimate the distance between nuclei for specific conditions of the methane-operating gas-expansion machine [3]. The technical parameters of the gas-expansion machine (for which calculations were performed) are: cylinder diameter 42 mm, stroke 66 mm, frequency of shaft rotation 980 rpm, coefficient of excess clearance 0.25, and mass flow rate of methane 32.2 kg/h (at the initial parameters 300 K and 4 MPa). For this gas-expansion machine the time of one shaft revolution is $6.12 \cdot 10^{-2}$ sec and the mean time in which the piston passes a distance of 1 cm is $\bar{\tau} = 4.64 \cdot 10^{-3}$ sec. The number of nuclei formed in a volume of 1 cm^3 during this time is $n = I\bar{\tau}$ cm^{-3} and the mean distance between nuclei is $\bar{L} = 1/n^{1/3}$ cm. Figure 2 shows the dependences of the mean distance between nuclei $\bar{L}$ and the ratio $\bar{l}/\bar{L}$ on the nucleation rate; in the calculations we took that $\bar{l} = 10^{-6}$ cm in all cases. It is seen from Fig. 2 that at $I \le 10^{15}$ cm$^{-3}\cdot$sec^{-1} the ratio $\bar{l}/\bar{L}$ does not exceed 0.02 and at I equal to 10^{19} cm$^{-3}\cdot$sec^{-1} or higher, $\bar{l}/\bar{L}$ has an order of unity. The estimates showed that for specific conditions of the problem being solved the value of I does not exceed 10^5–10^{10} cm$^{-3}\cdot$sec^{-1}, i.e., values of $\bar{l}/\bar{L}$ are not high. Higher values of I are given in the figure only to illustrate its effect on the ratio $\bar{l}/\bar{L}$.

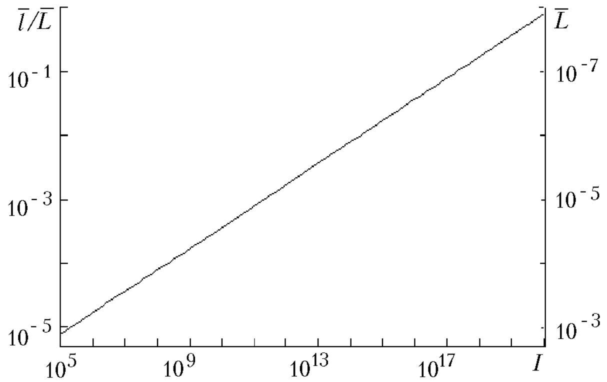

Fig. 2. Dependence of $\bar{L}$ (m) and the ratio $\bar{l}/\bar{L}$ on the nucleation rate.

Let the mean distance between the growing droplets in vapor be much larger than $\bar{l}$. Vapor at a distance from the droplet surface, which exceeds $\bar{l}$ by an order, can be considered as continuum. The formula for heat transfer in continuum (at distances from the droplet exceeding $\bar{l}$) has the form

$$rG_{\mathrm{v}} = \pi\delta^2\alpha\,(T_{\mathrm{b}} - T_{\infty})\,, \tag{6}$$

We consider that temperature T_{b} at a distance from the droplet surface, which is of the same order as the mean-free path of vapor molecules, is approximately equal to the temperature of the droplet liquid phase T_{d}. To allow for the effect of the Knudsen number, following the concepts used in [6] and taking that Nu = 2, we rewrite formula (6) in the form

$$rG_{\mathrm{v}} = \pi\delta^2\,(2\lambda_{\mathrm{v}}/\delta)\,(T_{\mathrm{d}} - T_{\infty})/\gamma\,, \tag{7}$$

where, according to the Sherman formula [6],

$$\gamma = 1 + b\,\mathrm{Kn}\,\beta^{-1}\,. \tag{8}$$

Here b is the dimensionless constant and $\beta \approx 1$ is the coefficient of thermal accommodation (in our case for methane molecules).

For monatomic gases it is suggested to take b = 3.75 and for two-atomic b = 3.17. For pentatomic gas — methane — the value of b must be lower. In what follows, we approximately take for methane b = 3. Since $G_{\mathrm{v}} = \pi\delta^2 g_{\mathrm{v}}$, for further calculation of quasi-stationary droplet growth we can use the differential equation of material balance of a growing droplet in the following form:

$$\rho_{\mathrm{liq}} d\,(\pi\delta^3/6)\;d\tau = \pi\delta^2 g_{\mathrm{v}}\,. \tag{9}$$

From Eqs. (7)–(9) we have

$$\frac{d\delta^2}{d\tau} = (1 + b/\delta)^{-1}\cdot 4\cdot\frac{2\lambda_{\mathrm{v}}}{r\rho_{\mathrm{liq}}}\,(T_{\mathrm{d}} - T_{\infty}) = (1 + b/\delta)^{-1}\cdot 4Q/\rho_{\mathrm{liq}}\,, \tag{10}$$

where $Q = g_{\mathrm{v}}\delta = 2\lambda_{\mathrm{v}}(T_{\mathrm{d}} - T_{\infty})/r$; we can take Q = const with variation of the droplet diameter [10] (in the case of constant pressure and temperature of vapor). After integration of (10) and transformations the equation for the droplet diameter takes on the form

$$\delta = -\,4b\bar{l} + \sqrt{(4b\bar{l})^2 + (4Q\tau/\rho_{\mathrm{liq}}) + \delta_0^2 + 4b\bar{l}\delta_0}\;. \tag{11}$$

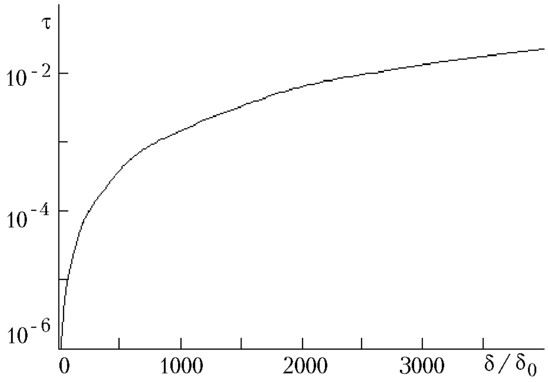

Fig. 3. Dependence of the time of condensation growth of a methane droplet on the relative diameter δ/δ_0 at $p = 8\cdot10^5$ Pa, $T = 138$ K, $\bar{l} = 2.9\cdot10^{-9}$ m, $Q = 5\cdot10^{-7}$ kg/(m·sec), and $\delta_0 = 3\cdot10^{-9}$ m.

An equation of type (11) with addition terms that are of no importance in condensation of pure methane vapor (or a mixture of methane with a small amount of nitrogen) was obtained in [11] for condensation of a small admixture of mercury vapor mixed with nitrogen. In the latter case, an important role is played by diffusion resistance to delivery of mercury vapor to the droplet surface. Formula (11) holds for calculation of the diameter of both large and small droplets.

It follows from expression (11) that at elevated values of the Knudsen number (small δ) the droplet diameter increases linearly with time, whereas at low values of this number the square of the droplet diameter increases linearly [10, 12]. Figure 3 presents the dependence of the time of methane droplet growth on the relative diameter δ/δ_0, which is calculated by formula (11) at $\bar{l} = 2.9\cdot10^{-9}$ m. The results show that small droplets grow very quickly compared with larger droplets. The time in which a droplet reaches a size of $3\cdot10^{-7}$ m ($\delta/\delta_0 = 100$) is only $1.8\cdot10^{-5}$ sec, whereas the time of droplet growth to $3\cdot10^{-6}$ and $1.2\cdot10^{-5}$ m is $1.2\cdot10^{-3}$ and $2.2\cdot10^{-2}$ sec, respectively.

Thermal coagulation of nuclei and droplets that is determined by Brownian motion can occur [6, 12]. However, the estimates showed that coagulation of nuclei under the conditions of the gas-expansion machine is expressed weakly (in contrast to coagulation of droplets in turbulent and reversible flows and in thermal coagulation during relatively large time intervals). We also note that vapor condensation can occur not only on spontaneously arising nuclei (homogeneous nucleation) but also on extraneous sites of condensation — dust particles and other foreign small particles (heterogeneous nucleation) [6]. In the case of gas liquefaction, these may be particles of earlier condensed impurity gases — hydrocarbons, CO_2, etc. But the number of extraneous sites is usually insufficient to reach in condensation on them of the required completeness of gas liquefaction. The suggested calculation of gas cooling and condensation can also be used in the case of a sufficient number of these sites (in a simplified form without calculations corresponding to nucleation processes).

We consider special features of condensation on small droplets related to the effect of curvature on saturated vapor pressure. Growth of these droplets will be somewhat retarded due to a decrease in the droplet surface temperature and decrease in heat removal caused by it. However, as has already been mentioned, the effect of surface curvature on vapor elasticity for methane droplets can be neglected with an error of 1.02, i.e., already for droplets with a diameter of $5\cdot10^{-8}$ m (at vapor pressure 0.8 MPa). Thus, as a whole, the error in determination of the time of droplet growth in Fig. 3 is not large.

We should bear in mind that the appearing nuclei have, in principle, different diameters — from small to larger than $\delta_{n,cr}$. Condensation cannot take place on nuclei with diameters smaller than $\delta_{n,cr}$, since in this case, free energy of the droplet–vapor system would increase. Only at nuclei diameters $\delta_n \geq \delta_{n,cr}$ can condensation with decreasing free energy occur. However, the number of nuclei whose diameter exceeds $\delta_{n,cr}$ is relatively small. Dispersion does not exceed several percent [13]. Therefore, it was assumed that, under these conditions, all nuclei have the same diameter $\delta_{n,cr}$ and just on them condensation will take place and further growth of droplets will occur.

On appearance of condensation the flow in the gas-expansion machine becomes nonisentropic (entropy will increase), but in the presence of thermal insulation it becomes adiabatic. In the adiabat equation (1), heat release in phase transition is disregarded (friction heat is neglected). We take into account condensation internal heat release for an adiabatically insulated system, having included into the adiabat equation the corresponding $dq_{\mathrm{in}} > 0$

$$dq_{\mathrm{in}} = di - vdp\ . \tag{12}$$

Then, in calculation by the zone method the heat release in nucleation is $\Delta N_{\mathrm{n}i}\dfrac{\rho_{\mathrm{liq}}\pi\delta_{\mathrm{n}i}^{3}}{6}r_i dv$, J/kmole, where $\Delta N_{\mathrm{n}i} = I_i\Delta\tau_i$ is the increment of the number of nuclei that appeared in the zone in time $\Delta\tau_i$, per cubic meter; $\dfrac{\rho_{\mathrm{liq}}\pi\delta_{\mathrm{n}i}^{3}}{6}r_i$ is the heat release per nucleus, J. On nuclei formed in each zone droplets grow, which increase in diameter in subsequent zones. The number of droplets of one size or another in the zone is determined by the number of nuclei from which this generation of droplets flows. The diameter, temperature with account for the surface curvature, and mass and its increment are calculated for each generation of droplets. Heat liberated by the droplets of the generation, which grew on the considered time step, is calculated as the product of the number of these droplets per volume unit by the increment of their mass and specific heat of condensation at the temperature of the droplet surface. The account of heat liberated in formation of all droplets is determined by summation of the amount of heat over all generations of the droplets of the zone (step).

Thus, if, in addition to nuclei, droplets grow, heat release related to condensation growth of droplets is added to the heat of nucleation. We denote $dq_{\mathrm{in}} = Bdv$, where B is the condensation heat release per gas volume unit, which coincides in dimensions with pressure. The initial equation for the section has the form

$$Bdv = di - vdp = C_p dT - vdp\ .$$

After transformations we obtain a differential equation that models adiabatic gas expansion in the presence of condensation:

$$\frac{dT}{T} = A_{\mathrm{eff}}dp\,\frac{p - B}{p^2 - A_{\mathrm{eff}}Bp}\ . \tag{13}$$

Integrating (13), we obtain a solution that relates the parameters of state (provided that B = const, as will be assumed in calculation):

$$T_2 = T_1\left[\frac{p_2 - A_{\mathrm{eff}}B}{p_1 - A_{\mathrm{eff}}B}\right]^{(A_{\mathrm{eff}}-1)}\frac{p_2}{p_1}\ . \tag{14}$$

In the p–V coordinates this solution has the form

$$\frac{p_2 - A_{\mathrm{eff}}B}{p_1 - A_{\mathrm{eff}}B} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{1-A_{\mathrm{eff}}}} = \left(\frac{V_1}{V_2}\right)^{k_{\mathrm{eff}}}\ . \tag{15}$$

At $B = 0$, an ordinary expression for adiabat (1) follows from (13) and (14). When $B > 0$, first, as pressure decreases, temperature still decreases, then, on reaching the minimum point (at $B = p$) the temperature decrease is superseded by its increase if the value of B is preserved.

Using Eqs. (14) and (15), we can not only calculate the parameters of the medium in the gas-expansion machine in the presence of condensation heat release but also take into account the effect of friction and heat fluxes from the outside in such devices, having included the quantity B into the correction that corresponds to these factors.

TABLE 2. Versions of Calculation

Version No.	Parameters at the inlet to the gas-expansion machine				Parameters at the outlet from the gas-expansion machine				
	S, kJ/(kg·K)	T, K	p, MPa	S_{sup}	S, kJ/(kg·K)	T, K	p, MPa	S_{sup}	K_{liqf}
1	3.60	171.7	2.19	0.885	3.97	141	0.68	1.001	0.06
2	3.70	172.3	2.00	0.792	4.01	139	0.61	1.002	0.05
3	3.80	171.3	1.75	0.715	4.05	136	0.52	1.02	0.04
4	3.90	171.5	1.55	0.629	3.90	120	0.41	2.09	0.001

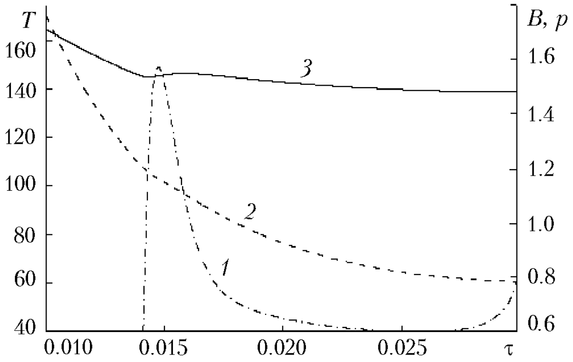

Fig. 4. Variation of temperature, methane pressure, and value of condensation heat release in volume unit B as a function of time of piston motion for the second version of calculation given in Table 2: 1) B, MJ/m^3 (MPa); 2) p, MPa; 3) T, K.

We made zone calculation of cooling and liquefaction of methane in the gas-expansion machine the technical parameters of which are given above. The increase in the cylinder volume at each step that is caused by piston motion is $V_{2i} = V_{1i} + \chi_i \Delta\tau_i$, where according to the main kinematic relations for the piston expansion machine [14]

$$\chi_i = \frac{V_c \pi}{T_{\text{fl}}} \left[\sin\,(2\pi\tau_i / T_{\text{fl}}) + \frac{\varepsilon}{2} \sin\,(4\pi\tau_i / T_{\text{fl}}) \right].$$

Calculation of methane cooling in the gas-expansion machine without supersaturation and, thus, without condensation, which was made at the initial parameters 300 K and 4 MPa, showed that the temperature that corresponds to the end of adiabatic expansion of methane is 215 K, which agrees with the result obtained in [3].

The main calculations are made in the gas-expansion machine for the cases of expansion with partial condensation of methane. Table 2 gives the corresponding initial parameters of state and values of the outlet parameters (at the end of expansion): temperature, pressure, entropy, values of supersaturation, and the liquefaction coefficient K_{liqf} for four versions. The liquefaction coefficient K_{liqf} is determined as the ratio of the formed mass of liquid methane to the mass of gas loaded to the gas-expansion machine. The versions are specified by the values of entropy at the inlet to the gas-expansion machine. Initial parameters of the computation versions are, correspondingly, chosen such that methane in the gas-expansion machine could reach the supersaturated vapor state. The time interval that corresponds to one step is taken as $\Delta\tau_i = 1 \cdot 10^{-6}$ sec; in this case, temperature variation at one step was not more than 0.001 K. A decrease in this interval, starting from $1 \cdot 10^{-5}$ sec, did not lead to noticeable changes in the computation results.

In all cases, the heat of nucleation per volume unit does not exceed 0.02 Pa, i.e., it is small compared with heat release due to droplet growth. Once the volumetric condensation heat release B exceeds the current pressure, a decrease in temperature, as has already been mentioned, gives way to its increase until the excess is maintained. These processes are illustrated in Fig. 4 for the second version of calculation. For the first and third versions similar graphs were obtained. Once the value of B becomes smaller than the current pressure, the medium temperature again begins to decrease (Fig. 4). From the moment the medium temperature begins to increase, the difference between the droplet

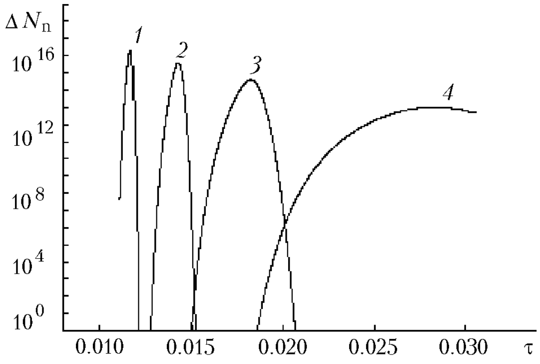

Fig. 5. Variation of the increment of the number of nuclei in the gas-expansion machine (per cubic meter) as a function of time of piston motion: 1, 2, 3, and 4 correspond to the versions of calculation given in Table 2.

and medium temperatures $T_d - T_\infty$, the corresponding heat removal from droplets, the growth of their size, and condensation heat release decrease. Calculations showed that the difference $T_d - T_\infty$ continues to decrease almost until the time instant that corresponds to the end of gas expansion.

At the initial parameters adopted for the fourth version, the process of isentropic gas cooling (until the origination of supersaturation) occupies a larger time interval than in other versions. Nucleation begins later (Fig. 5), the liquefaction process does not succeed in developing, and high supersaturation is maintained at the outlet. In the first three versions, droplets with different diameters appear in the gas-expansion machine earlier and their quantity is much larger than in the fourth version. In the last version, a noticeable number of droplets is fixed only to the time instant close to the end of piston motion toward gas expansion.

For the versions considered, the diameters of droplets at the end of gas expansion do not exceed several micrometers (in the first versions they are larger). The number of droplets per cubic meter decreases sharply with deviation of diameter values from a step-mean value.

Thus, at close temperatures of methane at the inlet to the gas-expansion machine, the liquefaction coefficient is the higher the lower the initial value of entropy. Results of the calculation illustrate the processes occurring in the piston expansion machine in the presence of the partially liquefied gas and allow a comparative analysis of different versions. This approach can be used in solution of specific engineering problems of cooling and liquefaction of other gases, e.g., helium and hydrogen, and gas mixtures, e.g., natural gas (which contains nitrogen as a basic noncondensing impurity).

An analysis of the processes of gas cooling and liquefaction with decrease in pressure and temperature at the outlet from the gas-expansion machine, in the heat exchanger, and then in the Joule–Thomson throttle (the final element in the liquefaction installation) is also of interest.

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NOTATION

$A = (k - 1)/k$; b, the dimensionless constant in the Sherman formula; B, condensation heat release per volume unit, J/m^3 (Pa); C_p and C_V, heat capacities at constant pressure and volume, respectively, J/(kg·deg); d_m, molecule diameter, m; G_v, vapor flow to the droplet surface, kg/sec; g_v, vapor-flow density near the droplet surface, kg/(m^2·sec); i, mole enthalpy, J/kmole; I, nucleation rate, cm^{-3}·sec^{-1}; k, adiabatic index of the ideal gas; $k_B = 1.38 \cdot 10^{-23}$, Boltzmann constant, J/K; $\mathrm{Kn} = \bar{l}/(\delta/2)$, Knudsen number; k_{liqf}, degree of gas liquefaction; $\bar{l}$, mean free path of a molecule, m; $\bar{L}$, mean distance between nuclei, m; $n = p/(k_B T)$, number of vapor molecules per volume unit, m^{-3}; $\mathrm{Nu} = \alpha\delta/\lambda_v$, Nusselt number for a droplet; ΔN_{ni}, increment of the number of nuclei per volume unit in time $\Delta\tau_i$, m^{-3}; p, pressure, MPa; q_{in}, heat release in gas condensation, J/kmole; Q, parameter in calculation of the velocity of droplet growth, kg/(m·sec); r, specific heat of phase transition, kJ/kg; $R = 8.31 \cdot 10^3$, universal gas constant, J/(kmole·K); S, entropy, kJ/(kg·K); S_{sup}, supersaturation; T, temperature, K; T_{fl}, time of one revolution of the flywheel, sec; v, molar volume, m^3/kmole; V, volume, m^3; ΔV_i, variation of the zone volume corresponding to one step, m^3; V_c, volume of the cylinder of the gas-expansion machine, m^3; α, heat-transfer coefficient, W/(m^2·deg); β, coefficient of thermal accommodation; δ, diameter (of a nucleus or droplet), m; ε, ratio of the crank radius to the length of the connecting rod; χ_i, velocity of gas volume growth behind the moving piston at time instant τ_i, m^3/sec; λ_v, thermal conductivity of vapor, W/(m·K); μ, molecular mass, kg/kmole; ρ, density, kg/m^3; σ, surface tension, N/m; τ, time, sec; $\Delta\tau_i$, time of one step, sec. Indices: 0, initial value; 1 and 2, inlet to and outlet from the zone; ∞, at a large distance from the droplet; in, internal; b, on the boundary that is at a distance from the surface droplet of an order of the mean-free path of vapor molecules; liq, liquid; n, nucleus; d, droplet; cr, critical value; m, molecule; fl, flywheel; v, vapor; f, flat surface; c, cylinder; eff, effective value corresponding to real gas; i, number of a step; s, saturation state; sup, supersaturation state; δ, curve surface corresponding to the diameter δ; overbar, mean value of the quantity; liqf, liquefaction.

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